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INTERNATIONAL STANDARD

Floating-point arithmetic



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FLOATING-POINT ARITHMETIC

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Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

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IEEE Std 754™-2019
(Revision of IEEE Std 754-2008)

IEEE Standard for Floating-Point Arithmetic

Sponsor

Microprocessor Standards Committee
of the
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Approved 13 June 2019

IEEE-SA Standards Board

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Abstract: This standard specifies interchange and arithmetic formats and methods for binary and decimal floating-point arithmetic in computer programming environments. This standard specifies exception conditions and their default handling. An implementation of a floating-point system conforming to this standard may be realized entirely in software, entirely in hardware, or in any combination of software and hardware. For operations specified in the normative part of this standard, numerical results and exceptions are uniquely determined by the values of the input data, sequence of operations, and destination formats, all under user control.

Keywords: arithmetic, binary, computer, decimal, exponent, floating-point, format, IEEE 754™, interchange, NaN, number, rounding, significand, subnormal.

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IEEE Introduction

This introduction is not part of IEEE Std 754-2019, IEEE Standard for Floating-Point Arithmetic.

This standard is a product of the Floating-Point Working Group of, and sponsored by, the Microprocessor Standards Committee of the IEEE Computer Society.

This standard provides a discipline for performing floating-point computation that yields results independent of whether the processing is done in hardware, software, or a combination of the two. For operations specified in the normative part of this standard, numerical results and exceptions are uniquely determined by the values of the input data, the operation, and the destination, all under user control.

This standard defines a family of commercially feasible ways for systems to perform binary and decimal floating-point arithmetic. Among the desiderata that guided the formulation of this standard were:

a) Facilitate movement of existing programs from diverse computers to those that adhere to this standard as well as among those that adhere to this standard.

b) Enhance the capabilities and safety available to users and programmers who, although not expert in numerical methods, might well be attempting to produce numerically sophisticated programs.

c) Encourage experts to develop and distribute robust and efficient numerical programs that are portable, by way of minor editing and recompilation, onto any computer that conforms to this standard and possesses adequate capacity. Together with language controls it should be possible to write programs that produce identical results on all conforming systems.

d) Provide direct support for
   - execution-time diagnosis of anomalies
   - smoother handling of exceptions
   - interval arithmetic at a reasonable cost.

e) Provide for development of
   - common elementary functions such as *exp* or *cos*
   - high precision (multiword) arithmetic
   - coupled numerical and symbolic algebraic computation.

f) Enable rather than preclude further refinements and extensions.

In programming environments, this standard is also intended to form the basis for a dialog between the numerical community and programming language designers. It is hoped that language-defined methods for the control of expression evaluation and exceptions might be defined in coming years, so that it will be possible to write programs that produce identical results on all conforming systems. However, it is recognized that utility and safety in languages are sometimes antagonists, as are efficiency and portability.

Therefore, it is hoped that language designers will look on the full set of operation, precision, and exception controls described here as a guide to providing the programmer with the ability to portably control expressions and exceptions. It is also hoped that designers will be guided by this standard to provide extensions in a completely portable way.

Informative annexes provide additional information – Annex A lists bibliographical resources, Annex B suggests programming environment features for debugging support, and Annex C lists all references to the operations of the standard.

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Floating-Point Arithmetic

1. Overview

1.1 Scope

This standard specifies formats and operations for floating-point arithmetic in computer systems. Exception conditions are defined and handling of these conditions is specified.

1.2 Purpose

This standard provides a method for computation with floating-point numbers that will yield the same result whether the processing is done in hardware, software, or a combination of the two. The results of the computation will be identical, independent of implementation, given the same input data. Errors, and error conditions, in the mathematical processing will be reported in a consistent manner regardless of implementation.

1.3 Inclusions

This standard specifies:

- Formats for binary and decimal floating-point data, for computation and data interchange.
- Addition, subtraction, multiplication, division, fused multiply add, square root, compare, and other operations.
- Conversions between integer and floating-point formats.
- Conversions between different floating-point formats.
- Conversions between floating-point formats and external representations as character sequences.
- Floating-point exceptions and their handling, including data that are not numbers (NaNs).

1.4 Exclusions

This standard does not specify:

- Formats of integers.
- Interpretation of the sign and significand fields of NaNs.

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1.5 Programming environment considerations

This standard specifies floating-point arithmetic in two radices, 2 and 10. A programming environment may conform to this standard in one radix or in both.

This standard does not define all aspects of a conforming programming environment. Such behavior should be defined by a programming language definition supporting this standard, if available, and otherwise by a particular implementation. Some programming language specifications might permit some behaviors to be defined by the implementation.

Language-defined behavior should be defined by a programming language standard supporting this standard. Then all implementations conforming both to this floating-point standard and to that language standard behave identically with respect to such language-defined behaviors. Standards for languages intended to reproduce results exactly on all platforms are expected to specify behavior more tightly than do standards for languages intended to maximize performance on every platform.

Because this standard requires facilities that are not currently available in common programming languages, the standards for such languages might not be able to fully conform to this standard if they are no longer being revised. If the language can be extended by a function library or class or package to provide a conforming environment, then that extension should define all the language-defined behaviors that would normally be defined by a language standard.

Implementation-defined behavior is defined by a specific implementation of a specific programming environment conforming to this standard. Implementations define behaviors not specified by this standard nor by any relevant programming language standard or programming language extension.

Conformance to this standard is a property of a specific implementation of a specific programming environment, rather than of a language specification.

However a language standard could also be said to conform to this standard if it were constructed so that every conforming implementation of that language also conformed automatically to this standard.

1.6 Word usage

In this standard three words are used to differentiate between different levels of requirements and optionality, as follows:

- **may** indicates a course of action permissible within the limits of the standard with no implied preference (“may” means “is permitted to”)
- **shall** indicates mandatory requirements strictly to be followed in order to conform to the standard and from which no deviation is permitted (“shall” means “is required to”)
- **should** indicates that among several possibilities, one is recommended as particularly suitable, without mentioning or excluding others; or that a certain course of action is preferred but not necessarily required; or that (in the negative form) a certain course of action is deprecated but not prohibited (“should” means “is recommended to”).

Further:

- **might** indicates the possibility of a situation that could occur, with no implication of the likelihood of that situation (“might” means “could possibly”)
- **see** followed by a number is a cross-reference to the clause or subclause of this standard identified by that number
- **NOTE** introduces text that is informative (that is, is not a requirement of this standard).

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2. Definitions, abbreviations, and acronyms

2.1 Definitions

For the purposes of this standard, the following terms and definitions apply.

applicable attribute: The value of an attribute governing a particular instance of execution of a computational operation of this standard. Languages specify how the applicable attribute is determined.

arithmetic format: A floating-point format that can be used to represent floating-point operands or results for the operations of this standard.

attribute: An implicit parameter to operations of this standard, which a user might statically set in a programming language by specifying a constant value. The term attribute might refer to the parameter (as in "rounding-direction attribute") or its value (as in "roundTowardZero attribute").

basic format: One of five floating-point representations, three binary and two decimal, whose encodings are specified by this standard, and which can be used for arithmetic. One or more of the basic formats is implemented in any conforming implementation.

biased exponent: The sum of the exponent and a constant (bias) chosen to make the biased exponent's range non-negative.

binary floating-point number: A floating-point number with radix two.

block: A language-defined syntactic unit for which a user can specify attributes. Language standards might provide means for users to specify attributes for blocks of varying scopes, even as large as an entire program and as small as a single operation.

canonical encoding: A preferred encoding of a floating-point representation in a format. "Canonical encoding" also applies to declets, significands of finite numbers, infinities, and NaNs, especially in decimal formats.

cohort: The set of all floating-point representations that represent a given floating-point number in a given floating-point format. In this context −0 and +0 are considered distinct and are in different cohorts.

computational operation: An operation that produces floating-point results or that might signal floating-point exceptions. Computational operations produce results in floating-point or other destination formats by rounding them to fit if necessary.

correct rounding: This standard's method of converting an infinitely precise result to a floating-point number, as determined by the applicable rounding direction. A floating-point number so obtained is said to be correctly rounded.

decimal floating-point number: A floating-point number with radix ten.

declet: An encoding of three decimal digits into ten bits using the densely packed decimal encoding scheme. Computational operations accept all 1024 possible declets in operands. Most computational operations produce only the 1000 canonical declets.

denormalized number: *See:* **subnormal number.**

destination: The location for the result of an operation upon one or more operands. A destination might be either explicitly designated by the user or implicitly supplied by the system (for example, intermediate results in subexpressions or arguments for procedures). Even though some languages place the results of intermediate calculations in destinations beyond the user's control, this standard defines the result of an operation in terms of that destination's format and the operands' values.

dynamic mode: An optional method of dynamically setting attributes by means of operations of this standard to set, test, save, and restore them.

exception: An event that occurs when an operation on some particular operands has no outcome suitable for every reasonable application. That operation might signal an exception by invoking default exception handling or alternate exception handling. Exception handling might signal further exceptions. Recognize that *event*, *exception*, and *signal* are defined in diverse ways in different programming environments.

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