Guide 98-5

# Guide to the expression of uncertainty in measurement —

Part 5:
**Examples**

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Given that ISO/IEC Guide 98-5 is identical in content to JCGM GUM-5, the decimal symbol is a point on the line in the English version and a comma on the line in the French version.

Annex ZZ has been appended to provide a list of corresponding ISO/IEC Guides and JCGM guidance documents for which equivalents are not given in the text.

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Joint Committee for Guides in Metrology

# Guide to the expression of uncertainty in measurement — Part 5: Examples

# Guide pour l'expression de l'incertitude de mesure — Partie 5: Exemples

JCGM GUM-5:2026

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# Foreword

In 1997 a Joint Committee for Guides in Metrology (JCGM), chaired by the Director of the Bureau International des Poids et Mesures (BIPM), was created by the seven international organizations that had originally in 1993 prepared the 'Guide to the expression of uncertainty in measurement' and the 'International vocabulary of basic and general terms in metrology'. The JCGM assumed responsibility for these two documents from the Technical Advisory Group 4 of the International Organization for Standardization (ISO/TAG4).

The Joint Committee is formed by the BIPM with the International Electrotechnical Commission (IEC), the International Federation of Clinical Chemistry and Laboratory Medicine (IFCC), the International Laboratory Accreditation Cooperation (ILAC), the International Organization for Standardization (ISO), the International Union of Pure and Applied Chemistry (IUPAC), the International Union of Pure and Applied Physics (IUPAP), and the Organisation Internationale De Métrologie Légale (OIML).

JCGM has two Working Groups. Working Group 1, 'Expression of uncertainty in measurement', has the task to promote the use of the 'Guide to the expression of uncertainty in measurement' and to prepare documents for its broad application. Working Group 2, 'Working Group on International vocabulary of basic and general terms in metrology', has the task to revise and promote the use of the International vocabulary of basic and general terms in metrology (VIM).

In 2008 the JCGM made available a slightly revised version (mainly correcting minor errors) of the 'Guide to the expression of uncertainty in measurement', labelling the document 'JCGM 100:2008'.

In 2017 the JCGM rebranded the documents in its portfolio that have been produced by Working Group 1 or are to be developed by that Group: the whole suite of documents is now known as the 'Guide to the expression of uncertainty in measurement' or 'GUM', and is concerned with the evaluation and expression of measurement uncertainty, as well as its application in science, trade, health, safety and other societal activities.

This part of the suite contains a collection of examples illustrating the methods for uncertainty evaluation described in the GUM.

This document has been prepared by Working Group 1 of the JCGM, and has benefited from detailed reviews undertaken by member organizations of the JCGM and National Metrology Institutes.

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# Introduction

The GUM suite of documents (parts overview in [14]) is concerned with the evaluation of uncertainty in measurement and how uncertainty is reported and used. Compact examples are given throughout the GUM suite that relate to specific areas of application and are intended to illustrate particular concepts. This document contains larger examples and case studies intended to illustrate the principles presented in the GUM suite for evaluating and reporting measurement uncertainty.

The examples are chosen to illustrate good practice in the evaluation of measurement uncertainty using the methods in the GUM, such as the JCGM 100:2008 uncertainty framework (GUF) [17, Definition 3.18] or the Monte Carlo method (MCM) [12,17] or alternative methods not (yet) covered by the guidance in the suite. In some examples, methods for uncertainty evaluation are compared to highlight aspects of their suitability for the presented case.

The context and measurement in the examples are described to the extent relevant; simplifications can have been made in the interest of presenting a concise case. The data used in the examples is, where possible, taken from real measurements. These data are not necessarily the best, representing the state-of-the-art or in any other way to be interpreted as such. Measurement procedures and practices are presented in a concise format and not intended to enable reproducing the measurement. Where appropriate, references are provided to documents describing these procedures or practices.

All examples were edited prior to inclusion in this document.

Results are generally reported in the manner described in JCGM 101:2008 [17]. However, more than the recommended one or two significant digits are often given here to facilitate comparison of the results obtained from the various approaches.

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# Guide to the expression of uncertainty in measurement — Part 5: Examples

## 0 Scope

**0.1** This document gives a number of examples, which are worked out in considerable detail to illustrate the principles presented in the Guide to the expression of uncertainty in measurement (GUM) for evaluating and reporting uncertainty in measurement. Together with the (shorter) examples included in the other parts of the GUM, they should enable users of the GUM to put these principles into practice in their own work.

**0.2** Because the examples are for illustrative purposes, they have by necessity been simplified (but not over-simplified). Moreover, because they and the numerical data used in them have been chosen mainly to demonstrate the principles of the GUM, neither they nor the data should necessarily be interpreted as describing real measurements, let alone the state of the art for a particular measurement. Although the data were used as provided, all calculations retained the full computational precision of the systems employed. Final and intermediate results have been rounded appropriately to suit the intended applications.

**0.3** The examples are used to demonstrate, among other:

- Using knowledge about a quantity to obtain an estimate of the quantity and its associated uncertainty;
- Propagation of uncertainty using the law of propagation of uncertainty (LPU) as in JCGM 100:2008 and JCGM 102:2011;
- Propagation of distributions using analytical and numerical methods, including the Monte Carlo method (MCM), as in JCGM 101:2008 and JCGM 102:2011;
- Applying law of propagation of uncertainty (LPU) and MCM for a univariate (scalar) and multivariate (vector) measurand;
- Applying LPU and MCM for independent and dependent input quantities;
- Using MCM to validate the results provided by LPU;
- Determining a coverage interval or a coverage region;
- Using measurement uncertainty in conformity assessment as in JCGM 106:2012;
- Reporting measurement results.

**0.4** An overview of the parts of the GUM suite is given in [14]. The conventions and notation applied in this document are summarised in Annex A. The symbols used throughout the document are explained in the text and in Annex B.

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