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Joint Committee for Guides in Metrology

Guide to the expression of uncertainty in measurement — Part 6: Developing and using measurement models

Guide pour l'expression de l'incertitude de mesure — Partie 6: Élaboration et utilisation des modèles de mesure

JCGM GUM-6:2020

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Foreword

In 1997 a Joint Committee for Guides in Metrology (JCGM), chaired by the Director of the Bureau International des Poids et Mesures (BIPM), was created by the seven international organizations that had originally in 1993 prepared the 'Guide to the expression of uncertainty in measurement' and the 'International vocabulary of basic and general terms in metrology'. The JCGM assumed responsibility for these two documents from the ISO Technical Advisory Group 4 (TAG4).

The Joint Committee is formed by the BIPM with the International Electrotechnical Commission (IEC), the International Federation of Clinical Chemistry and Laboratory Medicine (IFCC), the International Laboratory Accreditation Cooperation (ILAC), the International Organization for Standardization (ISO), the International Union of Pure and Applied Chemistry (IUPAC), the International Union of Pure and Applied Physics (IUPAP), and the International Organization of Legal Metrology (OIML).

JCGM has two Working Groups. Working Group 1, 'Expression of uncertainty in measurement', has the task to promote the use of the 'Guide to the expression of uncertainty in measurement' and to prepare documents for its broad application. Working Group 2, 'Working Group on International vocabulary of basic and general terms in metrology', has the task to revise and promote the use of the 'International vocabulary of basic and general terms in metrology' (the 'VIM').

In 2008 the JCGM made available a slightly revised version (mainly correcting minor errors) of the 'Guide to the expression of uncertainty in measurement', labelling the document 'JCGM 100:2008'. In 2017 the JCGM rebranded the documents in its portfolio that have been produced by Working Group 1 or are to be developed by that Group: the whole suite of documents became known as the 'Guide to the expression of uncertainty in measurement' or 'GUM'. This document, previously known as JCGM 103, Supplement 3 to the GUM, is the first to be published as a part of that portfolio, and is entitled and numbered accordingly.

The present guide is concerned with the development and use of measurement models, and supports the documents in the entire suite of JCGM documents concerned with uncertainty in measurement. The guide has been prepared by Working Group 1 of the JCGM, and has benefited from detailed reviews undertaken by member organizations of the JCGM and National Metrology Institutes.

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Introduction

A measurement model constitutes a relationship between the output quantities or measurands (the quantities intended to be measured) and the input quantities known to be involved in the measurement. There are several reasons for modelling a measurement. Models assist in developing a quantitative understanding of the measurement and in improving the measurement. A model enables values of the output quantities to be obtained given the values of the input quantities. Additionally, a model not only allows propagation of uncertainty from the input quantities to the output quantities; it also provides an understanding of the principal contributions to uncertainty. This document is accordingly concerned with the development of a measurement model and the practical use of the model.

One of the purposes of measurement is to assist in making decisions. The reliability of these decisions and the related risks depend on the values obtained for the output quantities and the associated uncertainties. In turn, these decisions depend on a suitable measurement model and the quality of information about the input quantities.

Although the development of a measurement model crucially depends on the nature of the measurement, some generic guidance on aspects of modelling is possible. A measurement model might be a straightforward mathematical relationship, such as the ideal gas law, or, at the other extreme, involve a sophisticated numerical algorithm for its evaluation, such as the detection of peaks in a signal and the determination of peak parameters.

A measurement model may take various forms: theoretical, empirical or hybrid (part-theoretical, part-empirical). It might have a single output quantity or more than one output quantity. The output quantity may or may not be expressed directly in terms of the input quantities. The quantities in the measurement model may be real-valued or complex-valued. Measurement models may be nested or multi-stage, in the sense that input quantities in one stage are output quantities from a previous stage, as occurs, for instance, in the dissemination of measurement standards or in calibration. Measurement models might describe time series of observations, including drift, and dynamic measurement. A measurement model may also take the form of a statistical model. In this document the concept ‘measurement model’ is intended in this broader meaning.

In developing or using a measurement model there are important choices to be made. The selection of a model that is adequate or fit for purpose is a key issue. Particularly for empirical models, there is choice of representation (or parametrization) of the families of functions concerned (polynomials, polynomial splines or rational functions, etc.). Certain choices can be far superior to others in their numerical behaviour when the model is implemented on a computer. The uncertainty arising from the choice of model is a necessary consideration.

In many disciplines, a basic measurement model requires extension to incorporate effects such as temperature corrections arising from the measurement to enable values for output quantities and the associated uncertainties to be obtained reliably.

Following the introduction in 1993 of the Guide to the expression of uncertainty in measurement, or GUM (also known as JCGM 100:2008), the practice of uncertainty evaluation has broadened to use a wider variety of models and methods. To reflect this, this Guide includes an introduction to statistical models for measurement modelling (clause 11) and additional guidance on modelling random variation in Annex C.

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Guide to the expression of uncertainty in measurement — Part 6: Developing and using measurement models

1 Scope

This document provides guidance on developing and using a measurement model and also covers the assessment of the adequacy of a measurement model. The document is of particular interest to developers of measurement procedures, working instructions and documentary standards. The model describes the relationship between the output quantity (the measurand) and the input quantities known to be involved in the measurement. The model is used to obtain a value for the measurand and an associated uncertainty. Measurement models are also used in, for example, design studies, simulation of processes, and in engineering, research and development.

This document explains how to accommodate in a measurement model the quantities involved. These quantities relate i) to the phenomenon or phenomena on which the measurement is based, that is, the measurement principle, ii) to effects arising in the specific measurement, and iii) to the interaction with the artefact or sample subject to measurement.

The guidance provided is organised in accordance with a work flow that could be contemplated when developing a measurement model from the beginning. This work flow starts with the specification of the measurand (clause 6). Then the measurement principle is modelled (clause 7) and an appropriate form of the model is chosen (clause 8). The basic model thus obtained is extended by identifying (clause 9) and adding (clause 10) effects arising from the measurement and the artefact or sample subject to measurement. Guidance on assessing the adequacy of the resulting measurement model is given in clause 12. The distinction between the basic model and the (complete) measurement model in the work flow should be helpful to those readers who already have a substantial part of the measurement model in place, but would like to verify that it contains all effects arising from the measurement so that it is fit for purpose.

Guidance on the assignment of probability distributions to the quantities appearing in the measurement model is given in JCGM 100:2008 and JCGM 101:2008. In clause 11, this guidance is supplemented by describing how statistical models can be developed and used for this purpose.

When using a measurement model, numerical problems can arise including computational effects such as rounding and numerical overflow. It is demonstrated how such problems can often be alleviated by expressing a model differently so that it performs well in calculations. It is also shown how a reformulation of the model can sometimes be used to eliminate some correlation effects among the input quantities when such dependencies exist.

Examples from a number of metrology disciplines illustrate the guidance provided in this document.

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2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML. Evaluation of measurement data — Guide to the expression of uncertainty in measurement. Joint Committee for Guides in Metrology, JCGM 100:2008.

BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML. Evaluation of measurement data — Supplement 1 to the 'Guide to the expression of uncertainty in measurement' — Propagation of distributions using a Monte Carlo method. Joint Committee for Guides in Metrology, JCGM 101:2008.

BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML. Evaluation of measurement data — Supplement 2 to the 'Guide to the expression of uncertainty in measurement' — Extension to any number of output quantities. Joint Committee for Guides in Metrology, JCGM 102:2011.

BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML. International vocabulary of metrology — Basic and general concepts and associated terms. Joint Committee for Guides in Metrology, JCGM 200:2012.

3 Terms and definitions

The terms and definitions of JCGM 100:2008, JCGM 101:2008, JCGM 102:2011 and JCGM 200:2012 apply.

ISO, IEC and IUPAC maintain terminological databases for use in standardization at the following addresses:

— IEC Electropedia: available at `http://www.electropedia.org`
— ISO Online Browsing Platform: available at `http://www.iso.org/obp`
— IUPAC Gold Book: available at `http://www.goldbook.iupac.org`

4 Conventions and notation

4.1 The conventions and notation in JCGM 100:2008, JCGM 101:2008 and JCGM 102:2011 are adopted. Principal symbols used throughout the document are explained in annex A. Other symbols and those appearing in examples are explained at first occurrence.

4.2 Most examples in this document contain numerical values rounded to a number of decimal digits appropriate to the application. Because of rounding there are often numerical inconsistencies among the values presented. An instance is the correlation coefficient of −0.817 in the example in 8.1.6. It is obtained from the *computer-held values* of two

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